Chapter 24

Sample Math Questions: Student-Produced Response

In this chapter, you will see examples of student-produced response math questions. This type of question appears in both the calculator and the no-calculator portions of the test. Student-produced response questions can come from any of the four areas covered by the SAT Math Test.

Student-Produced Response Strategies

Student-produced response questions do not have answer choices to select from. You must solve the problem and grid your answer on the answer sheet. There is a space to write your answer, and there are circles below to fill in for your answer. Use your written answer to make sure you fill in the correct circles. The filled-in circles are what determine how your answer is scored. You will not receive credit if you only write in your answer without filling in the circles.

REMEMBER

You must fill in the circles on the answer sheet in order to receive credit. You will not receive credit if you only write in your answer but don't fill in the circles.

Each grid has four columns. If your answer does not fill all four columns, leave the unneeded spaces blank. You may start your answer in any column as long as there is space to fill in the complete answer.

You should use many of the same test-taking strategies you used on the multiple-choice questions as for student-produced response questions, but here are a few additional tips to consider: First, remember that your answer must be able to fit in the grid on the answer sheet. If you solve the question and you found an answer that is negative or is greater than 9999, you should try to solve the problem a different way to find the correct answer. On some questions, your answer may include a dollar sign, a percent sign, or a degree symbol. You should not include these symbols in your answer, and as a reminder, the question will instruct you not to grid them.

When filling in fractions or decimals, keep a few things in mind. Answers cannot be mixed numbers. You must give your answer as an improper

REMEMBER

Answers cannot be mixed numbers. Give your answer as an improper fraction or as the equivalent decimal form. Thus, for instance, do *not* submit $3\frac{1}{2}$ as your answer. Instead, submit either $\frac{7}{2}$ or 3.5.

REMEMBER

You do not need to reduce fractions to their lowest terms as long as the fraction fits in the grid. You can save time and prevent calculation errors by giving your answer as an unreduced fraction.

fraction or as the equivalent decimal form. If your answer is a decimal with more digits than will fit in the grid, you must fill the entire grid with the most accurate value possible, either rounding the number or truncating it. Do not include a leading zero when gridding in decimals. For example, if your answer is $\frac{2}{3}$, you can grid 2/3, .666, or .667; however, 0.6, .66, and 0.67 would all be considered incorrect. Do not round up when truncating a number unless the decimal should be rounded up. For example, if the answer is $\frac{1}{3}$, .333 is an acceptable answer, but .334 is not. It is also not necessary to reduce fractions to their lowest terms as long as the fraction fits in the grid. If your answer is $\frac{6}{18}$, you do not need to reduce it to $\frac{1}{3}$. Giving your answer as an unreduced fraction (if it fits in the grid) can save you time and prevent simple calculation mistakes.

Make sure to read the question carefully and answer what is being asked. If the question asks for the number of thousands and the correct answer is 2 thousands, grid in 2 as the answer, not 2000. If the question asks for your answer to be rounded to the nearest tenth or hundredth, only a correctly rounded answer will be accepted.

Some student-produced response questions may have more than one correct answer. You only need to provide one answer. Do not attempt to grid in more than one answer. You should not spend your time looking for additional answers. Just like multiple-choice questions, there is no penalty for guessing on student-produced response questions. If you are not sure of the correct answer, make an educated guess. Try not to leave questions unanswered.

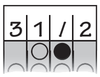
REMEMBER

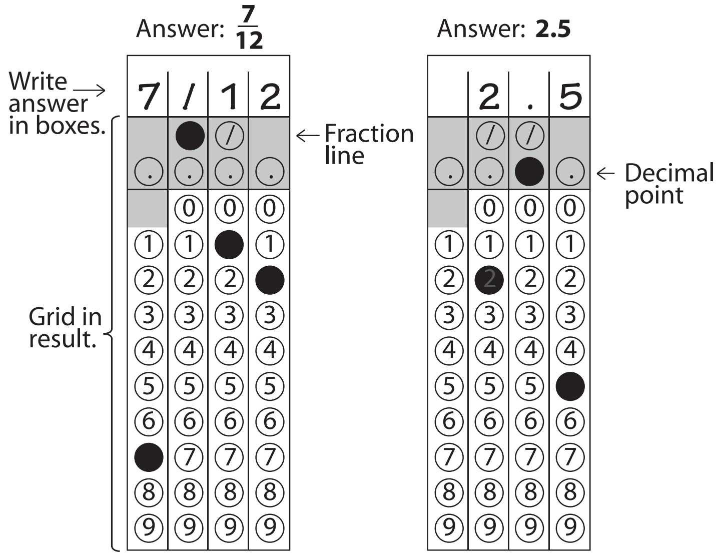
Carefully read the directions for the student-produced response questions now so you won't have to spend precious time doing so on test day.

The actual test directions for the student-produced response questions appear on the next page.

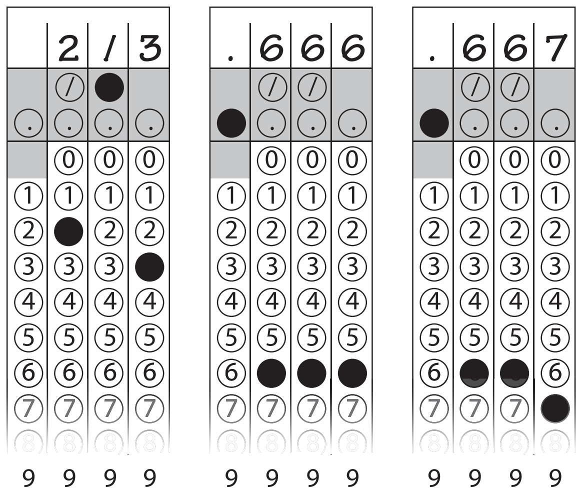
DIRECTIONS

For questions 31–38, solve the problem and enter your answer in the grid, as described below, on the answer sheet.

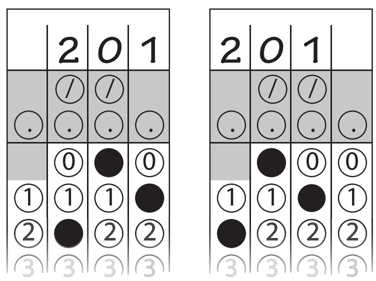
1. Although not required, it is suggested that you write your answer in the boxes at the top of the columns to help you fill in the circles accurately. You will receive credit only if the circles are filled in correctly.
2. Mark no more than one circle in any column.
3. No question has a negative answer.
4. Some problems may have more than one correct answer. In such cases, grid only one answer.
5. **Mixed numbers** such as $3\frac{1}{2}$ must be gridded as 3.5 or 7/2. (If  is entered into the grid, it will be interpreted as $\frac{31}{2}$, not $3\frac{1}{2}$.)
6. **Decimal answers:** If you obtain a decimal answer with more digits than the grid can accommodate, it may be either rounded or truncated, but it must fill the entire grid.



Acceptable ways to grid $\frac{2}{3}$ are:



Answer: **201** – either position is correct



NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Sample Questions: Student-Produced Response

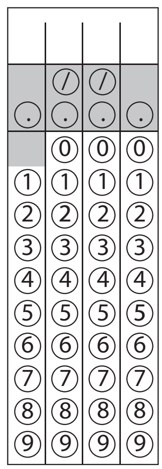
CALCULATOR PORTION

1

The table below classifies 103 elements as metal, metalloid, or nonmetal and as solid, liquid, or gas at standard temperature and pressure.

	Solids	Liquids	Gases	Total
Metals	77	1	0	78
Metalloids	7	0	0	7
Nonmetals	6	1	11	18
Total	90	2	11	103

What fraction of all solids and liquids in the table are metalloids?



Content: Problem Solving and Data Analysis

Key: .076, $\frac{7}{92}$

Objective: You must read information from a two-way table and determine the specific relationship between two categorical variables.

Explanation: There are 7 metalloids that are solid or liquid, and there are 92 total solids and liquids. Therefore, the fraction of solids and liquids that are metalloids is $\frac{7}{92}$.

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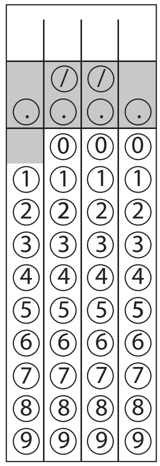
The denominator of the fraction will be the total number of solids and liquids, while the numerator will be the number of liquids and solids that are metalloids. Carefully retrieve that information from the table, and remember to shade in the circles that correspond to the answer.

2

A typical image taken of the surface of Mars by a camera is 11.2 gigabits in size. A tracking station on Earth can receive data from the spacecraft at a data rate of 3 megabits per second for a maximum of 11 hours each day. If 1 gigabit equals 1,024 megabits, what is the maximum number of typical images that the tracking station could receive from the camera each day?



Content: Problem Solving and Data Analysis

Key: 10

Objective: In this problem, students must use the unit rate (data-transmission rate) and the conversion between gigabits and megabits as well as conversions in units of time. Unit analysis is critical to solving the problem correctly, and the problem represents a typical calculation that would be done when working with electronic files and data-transmission rates.

Explanation: The tracking station can receive 118,800 megabits each day $\left(\frac{3 \text{ megabits}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times 11 \text{ hours}\right)$, which is about 116 gigabits each day $\left(\frac{118{,}800}{1{,}024}\right)$. If each image is 11.2 gigabits, then the number of images that can be received each day is $\frac{116}{11.2} \approx 10.4$. Since the question asks for the maximum number of typical images, rounding the answer down to 10 is appropriate because the tracking station will not receive a completed 11th image in one day.

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Unit analysis and conversion is an important skill on the SAT Math Test and features prominently on this question. It may help to write out the conversion, including the units, as illustrated here.

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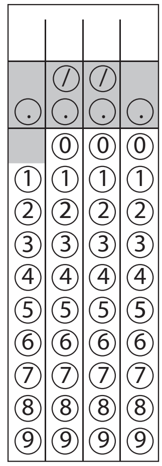
Consider whether rounding up or down is appropriate based on the question. Here, rounding 10.4 down to 10 is required to receive credit on this question since the question specifically asks for the maximum number of images that the tracking station can receive each day.

3

If $-\frac{9}{5} < -3t + 1 < -\frac{7}{4}$, what is one possible value of $9t - 3$?



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When you multiply an inequality by a negative number, remember to reverse the inequality signs.

REMEMBER

When entering your answer to this question, do not enter your answer as a mixed fraction. Rather, enter your answer as a decimal or an improper fraction.

Content: Heart of Algebra

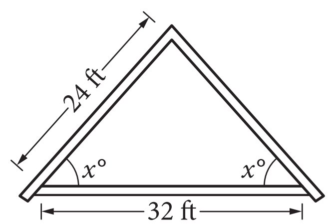
Key: Any value greater than $\frac{21}{4}$ and less than $\frac{27}{5}$

Objective: You should recognize the structure of the inequality to form a strategy to solve the inequality.

Explanation: Using the structure of the inequality to solve, you could note that the relationship between $-3t + 1$ and $9t - 3$ is that the latter is -3 multiplied by the former. Multiplying all parts of the inequality by -3 reverses the inequality signs, resulting in $\frac{27}{5} > 9t - 3 > \frac{21}{4}$ or rather $\frac{21}{4} < 9t - 3 < \frac{27}{5}$ when written with increasing values from left to right. Any value that is greater than $\frac{21}{4}$ and less than $\frac{27}{5}$ is correct.

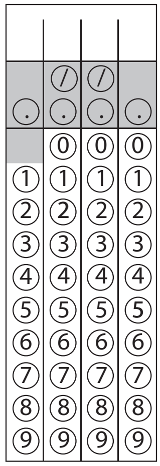
4

An architect drew the sketch below while designing a house roof. The dimensions shown are for the interior of the triangle.



Note: Figure not drawn to scale.

What is the value of cos x?



Content: Additional Topics in Math

Key: $\frac{2}{3}$, $\frac{4}{6}$, $\frac{8}{12}$, .666, .667

Objective: You must make use of properties of triangles to solve a problem.

Explanation: Because the triangle is isosceles, constructing a perpendicular from the top vertex to the opposite side will bisect the base and create two smaller right triangles. In a right triangle, the cosine of an acute angle is equal to the length of the side adjacent to the angle divided by the length of the hypotenuse. This gives $\cos x = \frac{16}{24}$, which can be simplified to $\cos x = \frac{2}{3}$. Note that $\frac{16}{24}$ cannot be entered into the answer grid, so this fraction must be reduced.

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The cosine of an acute angle is equal to the length of the side adjacent to the angle divided by the length of the hypotenuse. Learn to solve for sine, cosine, and tangent of an acute angle; this may be tested on the SAT.

SAMPLE QUESTION SET

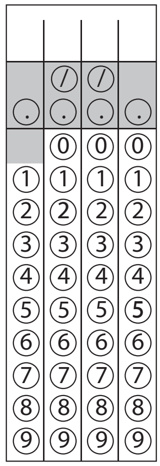
Questions 5 and 6 refer to the following information:

An international bank issues its Traveler credit cards worldwide. When a customer makes a purchase using a Traveler card in a currency different from the customer's home currency, the bank converts the purchase price at the daily foreign exchange rate and then charges a 4% fee on the converted cost.

Sara lives in the United States and is on vacation in India. She used her Traveler card for a purchase that cost 602 rupees (Indian currency). The bank posted a charge of \$9.88 to her account that included a 4% fee.

5

What foreign exchange rate, in Indian rupees per one U.S. dollar, did the bank use for Sara's charge? Round your answer to the nearest whole number.



Content: Problem Solving and Data Analysis

Key: 63

Objective: You must use the information in the problem to set up a ratio that will allow you to find the exchange rate.

Explanation: \$9.88 represents the conversion of 602 rupees plus a 4% fee on the converted cost. To calculate the original cost of the item in dollars, x, find $1.04x = 9.88$, $x = 9.5$. Since the original cost is \$9.50, to calculate the exchange rate r, in Indian rupees per one U.S. dollar: $9.50 \text{ dollars} \times \dfrac{r \text{ rupees}}{1 \text{ dollar}} = 602 \text{ rupees}$; solving for r yields approximately 63 rupees.

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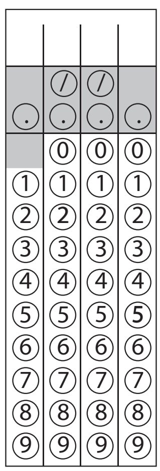
It is helpful to divide this question into two steps. First, calculate the original cost of Sara's purchase in dollars. Then, set up a ratio to find the exchange rate, keeping track of your units.

6

A bank in India sells a prepaid credit card worth 7500 rupees. Sara can buy the prepaid card using dollars at the daily exchange rate with no fee, but she will lose any money left unspent on the prepaid card. What is the least number of the 7500 rupees on the prepaid card Sara must spend for the prepaid card to be cheaper than charging all her purchases on the Traveler card? Round your answer to the nearest whole number of rupees.



Content: Problem Solving and Data Analysis

Key: 7212

Objective: You must set up an inequality to solve a multistep problem.

Explanation: Let d dollars be the cost of the 7500-rupee prepaid card. This implies that the exchange rate on this particular day is $\frac{d}{7500}$ dollars per rupee. Suppose Sara's total purchases on the prepaid card were r rupees. The value of r rupees in dollars is $\left(\frac{d}{7500}\right)r$ dollars. If Sara spent the r rupees on the Traveler card instead, she would be charged $1.04\left(\frac{d}{7500}\right)r$ dollars. To answer the question about how many rupees Sara must spend in order to make the Traveler card a cheaper option (in dollars) for spending the r rupees, you must set up the inequality $1.04\left(\frac{d}{7500}\right)r \geq d$. Rewriting both sides reveals $1.04\left(\frac{r}{7500}\right)d \geq (1)d$, from which you can infer $1.04\left(\frac{r}{7500}\right) \geq 1$. Dividing both sides by 1.04 and multiplying both sides by 7500 finally yields $r \geq 7{,}212$. Hence the least number of rupees Sara must spend for the prepaid card to be cheaper than the Traveler card is 7212.

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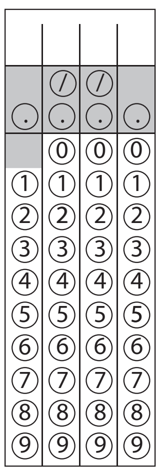
Another helpful way to think about this question is to keep in mind the fact that Sara will pay 7500 rupees for the prepaid card, regardless of how much money she leaves unspent. For the prepaid card to be cheaper than using the Traveler card, the Traveler card must end up costing Sara more than 7500 rupees. You can set up an inequality to calculate the least amount of purchases Sara needs to make using the Traveler card to exceed 7500 rupees. This value, when rounded to the nearest whole number, yields the correct answer.

Student-Produced Response Sample Questions

NO-CALCULATOR PORTION

7

If $a^2 + 14a = 51$ and $a > 0$, what is the value of $a + 7$?



Content: Passport to Advanced Math

Key: 10

Objective: You must use your knowledge of quadratic equations to determine the best way to efficiently solve this problem.

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This question, like many on the SAT Math Test, can be solved in a variety of ways. Use the method that will get you to the correct answer in the least amount of time. Knowing multiple approaches can also help in case you get stumped using one particular method.

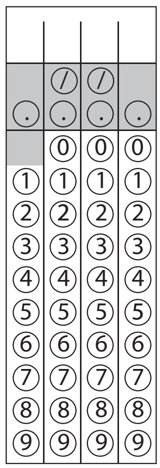
Explanation: There is more than one way to solve this problem. You can apply standard techniques by rewriting the equation $a^2 + 14a = 51$ as $a^2 + 14a - 51 = 0$ and then factoring. Since the coefficient of a is 14 and the constant term is -51, factoring requires writing 51 as the product of two numbers that differ by 14. This is $51 = (3)(17)$, which gives the factorization $(a + 17)(a - 3) = 0$. The possible values of a are -17 and 3. Since it is given that $a > 0$, it must be true that $a = 3$. Thus, the value of $a + 7$ is $3 + 7 = 10$.

You could also use the quadratic formula to find the possible values of a.

A third way to solve this problem is to recognize that adding 49 to both sides of the equation yields $a^2 + 14a + 49 = 51 + 49$, or rather $(a + 7)^2 = 100$, which has a perfect square on each side. Since $a > 0$, the solution to $a + 7 = 10$ is evident.

8

If $\frac{1}{2}x + \frac{1}{3}y = 4$, what is the value of $3x + 2y$?



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Always be on the lookout for shortcuts. On Question 8, for instance, examining the structure of the equation yields a very efficient solution.

Content: Heart of Algebra

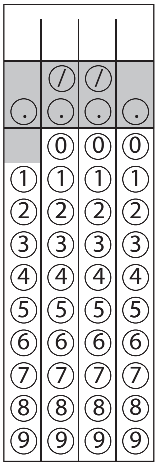
Key: 24

Objective: You must use the structure of the equation to efficiently solve the problem.

Explanation: Using the structure of the equation allows you to quickly solve the problem if you see that multiplying both sides of the equation by 6 clears the fractions and yields $3x + 2y = 24$.

9

What is one possible solution to the equation $\frac{24}{x+1} - \frac{12}{x-1} = 1$?



Content: Passport to Advanced Math

Key: 5, 7

Objective: You should seek the best solution method for solving rational equations before beginning. Searching for structure and common denominators will prove very useful at the onset and will help prevent complex computations that do not lead to a solution.

Explanation: In this problem, multiplying both sides of the equation by the common denominator $(x + 1)(x - 1)$ yields $24(x - 1) - 12(x + 1) = (x + 1)(x - 1)$. Multiplication and simplification then yields $12x - 36 = x^2 - 1$, or $x^2 - 12x + 35 = 0$. Factoring the quadratic gives $(x - 5)(x - 7) = 0$, so the solutions occur at $x = 5$ and $x = 7$, both of which should be checked in the original equation to ensure they are not extraneous. In this case, both values are solutions, and either is a correct answer.

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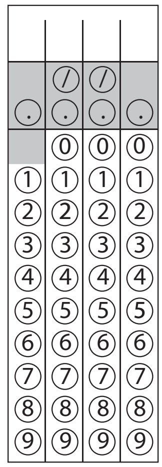
Eliminating fractions is often a good first step when asked to solve a rational equation. To eliminate the fractions in this equation, multiply both sides of the equation by the common denominator, which is $(x + 1)(x - 1)$.

10

$$x^2 + y^2 - 6x + 8y = 144$$

The equation of a circle in the xy-plane is shown above. What is the *diameter* of the circle?



Content: Additional Topics in Math

Key: 26

Objective: You must determine a circle property given the equation of the circle.

Explanation: Completing the square yields the equation $(x - 3)^2 + (y + 4)^2 = 169$, the standard form of an equation of the circle. Understanding this form results in the equation $r^2 = 169$, which when solved for r gives the value of the radius as 13. Diameter is twice the value of the radius; therefore, the diameter is 26.

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To solve Question 10, you must know that the standard form of the equation of a circle is $(x - a)^2 + (y - b)^2 = r^2$, where (a, b) is the center of the circle and r is the radius. You also must know how to complete a square.