Chapter 23

Sample Math Questions: Multiple-Choice

In the previous chapters, you learned about the four areas covered by the SAT Math Test. On the test, questions from the areas are mixed together, requiring you to solve different types of problems as you progress. In each portion, calculator and no calculator, you will first see multiple-choice questions and then student-produced response questions. This chapter will illustrate sample multiple-choice questions. These sample questions are divided into calculator and no-calculator portions just as they would be on the actual test.

Test-Taking Strategies

While taking the SAT Math Test, you may find that some questions are more difficult than others. Do not spend too much time on any one question. If you cannot answer a question in a reasonable amount of time, skip it and return to it after completing the rest of the section. It is important to practice this strategy because you do not want to waste time skipping around to find "easy" questions. Mark each question that you do not answer in your booklet so that you can easily go back to it later. In general, the multiple-choice and student-produced response sections are ordered with the easier questions first and the harder questions last, so keep in mind that the questions near the end of the test may take more time than the those at the beginning of the test.

Read each question carefully, making sure to pay attention to units and other key words and to understand exactly what information the question



It is important not to spend too much time on any question.
You will have on average a little less than a minute and a half per question on the calculator portion and a minute and fifteen seconds per question on the no-calculator portion.
If you can't solve a question in a reasonable amount of time, skip it (remembering to mark it in your booklet) and return to it later.

® REMEMBER

In general, the multiple-choice and student-produced response questions are ordered with the easier ones first and the harder ones last, so the later questions may take more time to solve than those at the beginning.

® REMEMBER

Knowing when to use a calculator is one of the skills that is assessed by the SAT Math Test. Keep in mind that some questions are actually solved more efficiently without the use of a calculator.

® REMEMBER

Never leave questions blank on the SAT, as there is no penalty for wrong answers. Thus, even if you're not sure of the correct answer, eliminate as many answer choices as you can and then guess from among the remaining ones. is asking for. You may find it helpful to underline key information in the problem, to draw figures to visualize the information given, or to mark key information on graphs and diagrams provided in the booklet.

When working through the test, remember to check your answer sheet to make sure you are filling in your answer on the correct row for the question you are answering. If your strategy involves skipping questions, it can be easy to get off track, so pay careful attention to your answer sheet.

On the calculator portion, keep in mind that using a calculator may not always be an advantage. Some questions are designed to be solved more quickly with manual calculations, so using a calculator may take more time. When using a calculator, always consider the reasonableness of an answer — this is the best way to catch mistakes that may have occurred when inputting your numbers.

Remember, there is no penalty for guessing on the SAT. Make your best guess for each question. Do not leave any questions blank on your answer sheet. For a question in which you are unsure of the correct answer, eliminating the answers that you know are wrong will give you a better chance of guessing the correct answer from the remaining choices.

On the no-calculator portion of the test, you have 25 minutes to answer 20 questions. This allows you an average of about 1 minute 15 seconds per question. Keep in mind that you should spend less time on easier questions so that you have more time available to spend on the more difficult ones.

Directions

The directions below will precede the no-calculator portion of the SAT Math Test. The directions for the calculator portion follow on the next page. The same references provided in the calculator portion of the SAT Math Test are also provided in the no-calculator portion of the test.

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Familiarize yourself with all test directions now so that you don't have to waste precious time on test day reading the directions.

Math Test – No Calculator

25 MINUTES, 20 QUESTIONS

Turn to Section 3 of your answer sheet to answer the questions in this section.

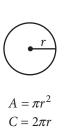
DIRECTIONS

For questions 1-15, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 16-20**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 16 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

NOTES

- 1. The use of a calculator is not permitted.
- 2. All variables and expressions used represent real numbers unless otherwise indicated.
- 3. Figures provided in this test are drawn to scale unless otherwise indicated.
- 4. All figures lie in a plane unless otherwise indicated.
- 5. Unless otherwise indicated, the domain of a given function f is the set of all real numbers x for which f(x) is a real number.

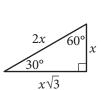
REFERENCE













 $A = \ell w$

 $A = \frac{1}{2}bh$

 $c^2 = a^2 + b^2$

Special Right Triangles







 $V = \pi r^2 h$



 $V = \frac{4}{3}\pi r^3$



 $V = \frac{1}{3}\pi r^2 h$



 $V = \frac{1}{3} \ell w h$

The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π .

The sum of the measures in degrees of the angles of a triangle is 180.

Math Test – Calculator

55 MINUTES, 38 QUESTIONS

Turn to Section 4 of your answer sheet to answer the questions in this section.

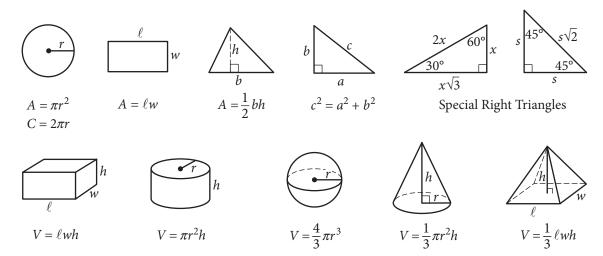
DIRECTIONS

For questions 1-30, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 31-38**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 31 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

NOTES

- 1. The use of a calculator is permitted.
- 2. All variables and expressions used represent real numbers unless otherwise indicated.
- 3. Figures provided in this test are drawn to scale unless otherwise indicated.
- 4. All figures lie in a plane unless otherwise indicated.
- 5. Unless otherwise indicated, the domain of a given function f is the set of all real numbers x for which f(x) is a real number.

REFERENCE



The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π .

The sum of the measures in degrees of the angles of a triangle is 180.

On the calculator portion of the test, you have 55 minutes to answer 38 questions. This allows you an average of about 1 minute 26 seconds per question. Keep in mind that you should spend less time on easier questions so that you have more time to spend on the more difficult ones.

Sample Questions: Multiple Choice — Calculator

1

The recommended daily calcium intake for a 20-year-old is 1,000 milligrams (mg). One cup of milk contains 299 mg of calcium and one cup of juice contains 261 mg of calcium. Which of the following inequalities represents the possible number of cups of milk m and cups of juice j a 20-year-old could drink in a day to meet or exceed the recommended daily calcium intake from these drinks alone?

- A) $299m + 261j \ge 1,000$
- B) 299m + 261j > 1,000

C)
$$\frac{299}{m} + \frac{261}{i} \ge 1,000$$

D)
$$\frac{299}{m} + \frac{261}{i} > 1,000$$

Content: Heart of Algebra

Key: A

Objective: You must identify the correct mathematical notation for an inequality to represent a real-world situation.

Explanation: Choice A is correct. Multiplying the number of cups of milk by the amount of calcium each cup contains and multiplying the number of cups of juice by the amount of calcium each cup contains gives the total amount of calcium from each source. You must then find the sum of these two numbers to find the total amount of calcium. Because the question asks for the calcium from these two sources to meet or exceed the recommended daily intake, the sum of these two products must be greater than or equal to 1,000.

Choice B is incorrect and may result from a misunderstanding of the meaning of inequality symbols as they relate to real-life situations. This answer does not allow for the daily intake to meet the recommended daily amount.

Choice C is incorrect and may result from a misunderstanding of proportional relationships. Here the wrong operation is applied, with the total amount of calcium per cup divided by the number of cups of each type of drink. These values should be multiplied.

Choice D is incorrect and may result from a combination of mistakes. The inequality symbol used allows the option to exceed, but not to meet, the recommended daily value, and the wrong operation may have been applied when calculating the total amount of calcium intake from each drink.

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On questions involving inequalities, pay close attention to whether or not the correct answer should include an equal sign.

2

A research assistant randomly selected 75 undergraduate students from the list of all students enrolled in the psychology-degree program at a large university. She asked each of the 75 students, "How many minutes per day do you typically spend reading?" The mean reading time in the sample was 89 minutes, and the margin of error for this estimate was 4.28 minutes. Another research assistant intends to replicate the survey and will attempt to get a smaller margin of error. Which of the following samples will most likely result in a smaller margin of error for the estimated mean time students in the psychology-degree program read per day?

- A) 40 randomly selected undergraduate psychology-degree program students
- B) 40 randomly selected undergraduate students from all degree programs at the college
- C) 300 randomly selected undergraduate psychology-degree program students
- D) 300 randomly selected undergraduate students from all degree programs at the college

Content: Problem Solving and Data Analysis

Key: C

Objective: You must first read and understand the statistics calculated from the survey. Then, you must apply your knowledge about the relationship between sample size and subject selection on margin of error.

Explanation: Choice C is correct. Increasing the sample size while randomly selecting participants from the original population of interest will most likely result in a decrease in the margin of error.

Choice A is incorrect and may result from a misunderstanding of the importance of sample size to a margin of error. The margin of error is likely to increase with a smaller sample size.

Choice B is incorrect and may result from a misunderstanding of the importance of sample size and participant selection to a margin of error. The margin of error is likely to increase due to the smaller sample size. Also, a sample of undergraduate students from all degree programs at the college is a different population than the original survey; therefore, the impact to the mean and margin of error cannot be predicted.

Choice D is incorrect. A sample of undergraduate students from all degree programs at the college is a different population than the original survey and therefore the impact to the mean and margin of error cannot be predicted.

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As discussed in Chapter 20, margin of error is affected by two factors: the variability in the data and the sample size. Increasing the size of the random sample provides more information and reduces the margin of error.

3

A company's manager estimated that the cost C, in dollars, of producing n items is C = 7n + 350. The company sells each item for \$12. The company makes a profit when the total income from selling a quantity of items is greater than the total cost of producing that quantity of items. Which of the following inequalities gives all possible values of n for which the manager estimates that the company will make a profit?

- A) n < 70
- B) n < 84
- C) n > 70
- D) n > 84

Content: Heart of Algebra

Key: C

Objective: You must interpret an expression or equation that models a real-world situation and be able to interpret the whole expression (or specific parts) in terms of its context.

Explanation: Choice C is correct. One way to find the correct answer is to create an inequality. The income from sales of n items is 12n. For the company to profit, 12n must be greater than the cost of producing n items; therefore, the inequality 12n > 7n + 350 can be used to model the context. Solving this inequality yields n > 70.

Choice A is incorrect and may result from a misunderstanding of the properties of inequalities. You may have found the number of items of the break-even point as 70 and used the incorrect notation to express the answer, or you may have incorrectly modeled the scenario when setting up an inequality to solve.

Choice B is incorrect and may result from a misunderstanding of how the cost equation models the scenario. If you use the cost of \$12 as the number of items n and evaluate the expression 7n, you will find the value of 84. Misunderstanding how the inequality relates to the scenario might lead you to think n should be less than this value.

Choice D is incorrect and may result from a misunderstanding of how the cost equation models the scenario. If you use the cost of \$12 as the number of items n and evaluate the expression 7n, you will find the value of 84. Misunderstanding how the inequality relates to the scenario might lead you to think n should be greater than this value.

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Remember to solve an inequality just as you would an equation, with one important exception. When multiplying or dividing both sides of an inequality by a negative number, you must reverse the direction of the inequality:

If -2x > 6, then x < -3.

4

At a primate reserve, the mean age of all the male primates is 15 years, and the mean age of all female primates is 19 years. Which of the following must be true about the mean age *m* of the combined group of male and female primates at the primate reserve?

- A) m = 17
- B) m > 17
- C) m < 17
- D) 15 < m < 19

Content: Problem Solving and Data Analysis

Key: D

Objective: You must evaluate the means for two separate populations in order to determine the constraints on the mean for the combined population.

Explanation: Choice D is correct. You must reason that because the mean of the males is lower than that of the females, the combined mean cannot be greater than or equal to that of the females, while also reasoning that because the mean of the females is greater than that of the males, the combined mean cannot be less than or equal to the mean of the males. Therefore, the combined mean must be between the two separate means.

Choice A is incorrect and results from finding the mean of the two means. This answer makes an unjustified assumption that there are an equal number of male and female primates.

Choice B is incorrect and results from finding the mean of the two means and misapplying an inequality to the scenario. This answer makes an unjustified assumption that there are more females than males.

Choice C is incorrect and results from finding the mean of the two means and misapplying an inequality to the scenario. This answer makes an unjustified assumption that there are more males than females.

5

A researcher wanted to know if there is an association between exercise and sleep for the population of 16-year-olds in the United States. She obtained survey responses from a random sample of 2,000 United States 16-year-olds and found convincing evidence of a positive association between exercise and sleep. Which of the following conclusions is well supported by the data?

- A) There is a positive association between exercise and sleep for 16-year-olds in the United States.
- B) There is a positive association between exercise and sleep for 16-year-olds in the world.
- C) Using exercise and sleep as defined by the study, an increase in sleep is caused by an increase of exercise for 16-year-olds in the United States.
- D) Using exercise and sleep as defined by the study, an increase in sleep is caused by an increase of exercise for 16-year-olds in the world.

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Question 4 does not require extensive calculation, or really any calculation at all.
Rather, it relies upon a solid understanding of mean along with careful reasoning. On the SAT, it pays to reason critically about the question before diving into calculations.

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When deciding what conclusions are supported by the data from a study or survey, ask yourself:

- 1. Was the sample of subjects in the study selected at random from the entire population in question? If so, the results can be generalized to the entire population in question. However, check to make sure that the conclusion is referring to the same population as that in the study.
- 2. Were the subjects randomly assigned to treatments? If so, conclusions about cause and effect can be drawn.

Content: Problem Solving and Data Analysis

Key: A

Objective: You must use information from a research study to evaluate whether the results can be generalized to the study population and whether a cause-and-effect relationship exists. To conclude a cause-and-effect relationship like the ones described in choices C and D, there must be a random assignment of participants to groups receiving different treatments. To conclude that the relationship applies to a population, participants must be randomly selected from that population.

Explanation: Choice A is correct. A relationship in the data can only be generalized to the population that the sample was drawn from.

Choice B is incorrect. A relationship in the data can only be generalized to the population that the sample was drawn from. The sample was from high school students in the United States, not from high school students in the entire world.

Choice C is incorrect. Evidence for a cause-and-effect relationship can only be established when participants are randomly assigned to groups who receive different treatments.

Choice D is incorrect. Evidence for a cause-and-effect relationship can only be established when participants are randomly assigned to groups who receive different treatments. Also, a relationship in the data can only be generalized to the population that the sample was drawn from. The sample was from high school students in the United States, not from high school students in the entire world.

6

A biology class at Central High School predicted that a local population of animals will double in size every 12 years. The population at the beginning of 2014 was estimated to be 50 animals. If P represents the population n years after 2014, then which of the following equations represents the class's model of the population over time?

- A) P = 12 + 50n
- B) P = 50 + 12n
- C) $P = 50(2)^{12n}$
- D) $P = 50(2)^{\frac{n}{12}}$

Content: Passport to Advanced Math

Key: D

Objective: You must identify the correct mathematical notation for an exponential relationship that represents a real-world situation.

Explanation: Choice D is correct. You first recognize that a population that doubles in size over equal time periods is increasing at an exponential rate. In a doubling scenario, an exponential growth model can be written in the form $y = a(2)^{\frac{n}{b}}$, where a is the initial population (that is, the population when n = 0) and b is the number of years it takes for the population to double in size. In this case, the initial population is 50, the number of animals at the

PRACTICE AT



A good strategy for checking your answer on Question 6 is to pick a number for n and test the answer choices. If n = 12, for instance, P should equal 100 (since after 12 years, the initial population of 50 should double to 100). Only choice D yields a value of 100 when you plug in 12 for n.

beginning of 2014. Therefore, a = 50. The text explains that the population will double in size every 12 years. Therefore, b = 12.

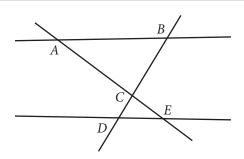
Choice A is incorrect and may result from a misunderstanding of exponential equations or of the context. This linear model indicates that the initial population is 12 animals and the population is increasing by 50 animals each year. However, this is not the case.

Choice B is incorrect and may result from a misunderstanding of exponential equations or of the scenario. This linear model indicates that the initial population is 50 animals and the population is increasing by 12 animals each year. However, this is not the case.

Choice C is incorrect. This exponential model indicates that the initial population is 50 animals and is doubling. However, the exponent 12*n* indicates that the population is doubling 12 times per year, not every 12 years. This is not the case.

[≸]REMEMBER

When a question explicitly states that a figure is *not* drawn to scale, avoid making unwarranted assumptions. Rely instead on your knowledge of mathematical properties and theorems.



Note: Figure not drawn to scale.

In the figure above, $\triangle ABC \sim \triangle EDC$. Which of the following must be true?

- A) $\overline{AE} \parallel \overline{BD}$
- B) $\overline{AE} \perp \overline{BD}$
- C) $\overline{AB} \parallel \overline{DE}$
- D) $\overline{AB} \perp \overline{DE}$

Content: Additional Topics in Math

Key: C

7

Objective: You must use spatial reasoning and geometric logic to deduce which relationship is true based on the given information. You must also use mathematical notation to express the relationship between the line segments.

Explanation: Choice C is correct. Given that $\triangle ABC$ is similar to $\triangle EDC$, you can determine that the corresponding $\angle BAC$ is congruent to $\angle CED$. The converse of the alternate interior angle theorem tells us that $\overline{AB} \parallel \overline{DE}$. (You can also use the fact that $\angle ABC$ and $\angle CDE$ are congruent to make a similar argument.)

Choice A is incorrect and may result from multiple misconceptions. You may have misidentified the segments as perpendicular and used the wrong notation to express this statement.

Choice B is incorrect and may result from using only the diagram and not considering the given information. The line segments appear to be perpendicular, but need not be, given the information provided.

Choice D is incorrect and may result from misunderstanding either the notation or the vocabulary of parallel and perpendicular lines. You may have incorrectly identified or notated parallel lines as perpendicular.

8

The function f is defined by $f(x) = 2x^3 + 3x^2 + cx + 8$ where c is a constant. In the xy-plane, the graph of f intersects the x-axis at the three points $(-4, 0), \left(\frac{1}{2}, 0\right)$, and (p, 0). What is the value of c?

- A) -18
- B) -2
- C) 2
- D) 10

Content: Passport to Advanced Math

Key: A

Objective: You could tackle this problem in many different ways, but the focus is on your understanding of the zeros of a polynomial function and how they are used to construct algebraic representations of polynomials.

Explanation: Choice A is correct. The given zeros can be used to set up an equation to solve for c. Substituting -4 for x and 0 for y yields -4c = 72, or c = -18. Alternatively, since -4, $\frac{1}{2}$, and p are zeros of the polynomial function, it follows that f(x) = (2x - 1)(x + 4)(x - p). Were this polynomial multiplied out, the constant term would be (-1)(4)(-9) = 4p. (We can grasp this without performing the full expansion.) Since it is given that this value is 8, it goes that 4p = 8 or, rather, p = 2. Substituting 2 for p in the polynomial function yields f(x) = (2x - 1)(x + 4)(x - 2), and after multiplying the factors, one finds that the coefficient of the x term, or the value of c, is -18.

Choice B is incorrect. This value may be the result of solving for p(p = 2) and then misunderstanding the relationship between the constants p and c in the equation.

Choice C is incorrect. This is the value of *p*, not *c*. Finding the value of *p* is an intermediate step to finding the value of *c*, but the value of *p* is not the final answer.

Choice D is incorrect. This value could be the result of an arithmetic error. Using the value of p(p = 2) and the other zeros, f(x) can be factored as f(x) = (2x-1)(x+4)(x-2). If the x terms in the product were erroneously found to be 14x and -4x, then combining like terms could result in this incorrect answer.

PRACTICE AT

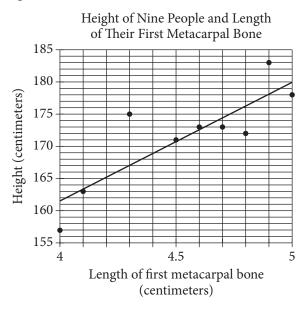
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When a question states that the graph of a function intersects the x-axis at specific points, this means that the dependent variable (f(x)) equals zero for the specified values of the independent variable (x). Applying this concept leads to the solution on Question 8.

SAMPLE QUESTION SET

Samples 9 to 11 refer to the following information:

The first metacarpal bone is located in the hand. The scatterplot below shows the relationship between the length of the first metacarpal bone and the height of 9 people. The line of best fit is also shown.



9

How many of the 9 people have an actual height that differs by more than 3 centimeters from the height predicted by the line of best fit?

- A) 2
- B) 4
- C) 6
- D) 9

Content: Problem Solving and Data Analysis

Key: B

Objective: You must read and interpret information from a data display.

Explanation: Choice B is correct. The people who have first metacarpal bones of length 4.0, 4.3, 4.8, and 4.9 centimeters have heights that differ by more than 3 centimeters from the height predicted by the line of best fit.

Choice A is incorrect. There are 2 people whose actual heights are more than 3 centimeters above the height predicted by the line of best fit. However, there are also 2 people whose actual heights are farther than 3 centimeters below the line of best fit.

Choice C is incorrect. There are 6 data points in which the absolute value between the actual height and the height predicted by the line of best fit is greater than 1 centimeter.

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Pay close attention to axis labels as well as to the size of the units on the two axes.

Choice D is incorrect. The data on the graph represent 9 different people; however, the absolute value of the difference between actual height and predicted height is not greater than 3 for all of the people.

10

Which of the following is the best interpretation of the slope of the line of best fit in the context of this problem?

- A) The predicted height increase in centimeters for one centimeter increase in the first metacarpal bone
- B) The predicted first metacarpal bone increase in centimeters for every centimeter increase in height
- C) The predicted height in centimeters of a person with a first metacarpal bone length of 0 centimeters
- D) The predicted first metacarpal bone length in centimeters for a person with a height of 0 centimeters

Content: Heart of Algebra

Key: A

Objective: You must interpret the meaning of the slope of the line of best fit in the context provided.

Explanation: Choice A is correct. The slope is the change in the vertical distance divided by the change in the horizontal distance between any two points on a line. In this context, the change in the vertical distance is the change in the predicted height of a person, and the change in the horizontal distance is the change in the length of his or her first metacarpal bone. The unit rate, or slope, is the increase in predicted height for each increase of one centimeter of the first metacarpal bone.

Choice B is incorrect. If you selected this answer, you may have interpreted slope incorrectly as run over rise.

Choice C is incorrect. If you selected this answer, you may have mistaken slope for the *y*-intercept.

Choice D is incorrect. If you selected this answer, you may have mistaken slope for the *x*-intercept.

11

Based on the line of best fit, what is the predicted height for someone with a first metacarpal bone that has a length of 4.45 centimeters?

- A) 168 centimeters
- B) 169 centimeters
- C) 170 centimeters
- D) 171 centimeters

Content: Problem Solving and Data Analysis

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Throughout the SAT MathTest, you will be asked to apply your knowledge of math principles and properties, such as slope, to specific contexts, such as the line of best fit in the scatterplot above. To do so requires that you possess a strong understanding of these math concepts.

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The answer choices on Question 11 are very close together. Thus, be very precise when examining the scatterplot to find the *y*-value that corresponds to an *x*-value of 4.45 on the line of best fit.

Objective: You must use the line of best fit to make a prediction. You must also demonstrate fluency in reading graphs and decimal numbers.

Explanation: Choice C is correct. You must notice that the scale of the *x*-axis is 0.1, and therefore the *x*-value of 4.45 is halfway between the unmarked value of 4.4 and the marked value of 4.5. You must then find the *y*-value on the line of best fit that corresponds with an *x*-value of 4.45, which is 170.

Choice A is incorrect. If you mistakenly find the point on the line between the *x*-values of 4.3 and 4.4, you will likely find a predicted metacarpal bone length of 168 centimeters.

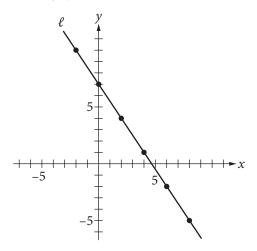
Choice B is incorrect. If you mistakenly find the point on the line that corresponds with an *x*-value of 4.4 centimeters, you will likely find a predicted height of approximately 169 centimeters.

Choice D is incorrect. If you mistakenly find the point on the line that corresponds with an *x*-value of 4.5 centimeters, you will likely find a predicted height of approximately 171 centimeters. You might also choose this option if you mistakenly use the data point that has an *x*-value closest to 4.45 centimeters.

Sample Questions: Multiple Choice — No Calculator

12

Line ℓ is graphed in the *xy*-plane below.



If line ℓ is translated up 5 units and right 7 units, then what is the slope of the new line?

- A) $-\frac{2}{5}$
- B) $-\frac{3}{2}$
- C) $-\frac{8}{9}$
- D) $-\frac{11}{14}$

Content: Heart of Algebra

Key: B

Objective: You must make a connection between the graphical form of a relationship and a numerical description of a key feature.

Explanation: Choice B is correct. The slope of a line can be determined by finding the difference in the *y*-coordinates divided by the difference in the *x*-coordinates for any two points on the line. Using the points indicated, the slope of line ℓ is $-\frac{3}{2}$. Translating line ℓ moves all the points on the line the same distance in the same direction, and the image will be a line parallel to ℓ . Therefore, the slope of the image is also $-\frac{3}{2}$.

Choice A is incorrect. This value may result from a combination of errors. You may have erroneously determined the slope of the new line by adding 5 to the numerator and adding 7 to the denominator in the slope of line ℓ and gotten the result (-3 + 5)/(-2 + 7).

Choice C is incorrect. This value may result from a combination of errors. You may have erroneously determined the slope of the new line by subtracting 5 from the numerator and subtracting 7 from the denominator in the slope of line ℓ .

Choice D is incorrect and may result from adding $\frac{5}{7}$ to the slope of line ℓ .

13

The mean number of students per classroom, y, at Central High School can be estimated using the equation y = 0.8636x + 27.227, where x represents the number of years since 2004 and $x \le 10$. Which of the following statements is the best interpretation of the number 0.8636 in the context of this problem?

- A) The estimated mean number of students per classroom in 2004
- B) The estimated mean number of students per classroom in 2014
- C) The estimated yearly decrease in the mean number of students per classroom
- D) The estimated yearly increase in the mean number of students per classroom

Content: Heart of Algebra

Key: D

Objective: You must interpret the slope of an equation in relation to the real-world situation it models. Also, when the models are created from data, you must recognize that these models only estimate the independent variable, *y*, for a given value of *x*.

Explanation: Choice D is correct. When an equation is written in the form y = mx + b, the coefficient of the *x*-term (in this case 0.8636) is the slope. The slope of this linear equation gives the amount that the mean number of students per classroom (represented by *y*) changes per year (represented by *x*).

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Your first instinct on this question may be to identify two coordinates on line ℓ , shift each of them over 5 and up 7, and then calculate the slope using the change in y over the change in x. While this will yield the correct answer, realizing that a line that is translated is simply shifted on the coordinate plane but retains its original slope will save time and reduce the chance for error. Always think critically about a question before diving into your calculations.

Choice A is incorrect and may result from a misunderstanding of slope and *y*-intercept. The *y*-intercept of the equation represents the estimated mean number of students per classroom in 2004.

Choice B is incorrect and may result from a misunderstanding of the limitations of the model. You may have seen that $x \le 10$ and erroneously used this statement to determine that the model finds the mean number of students in 2014.

Choice C is incorrect and may result from a misunderstanding of slope. You may have recognized that slope models the rate of change but thought that a slope of less than 1 indicates a decreasing function.

14

If
$$\frac{2}{a-1} = \frac{4}{y}$$
, and $y \ne 0$ where $a \ne 1$, what is y in terms of a?

A)
$$y = 2a - 2$$

B)
$$y = 2a - 4$$

C)
$$y = 2a - \frac{1}{2}$$

D)
$$y = \frac{1}{2}a + 1$$

Content: Passport to Advanced Math

Key: A

Objective: You must complete operations with multiple terms and manipulate an equation to isolate the variable of interest.

Explanation: Choice A is correct. Multiplying both sides of the equation by the denominators of the rational expressions in the equation gives 2y = 4a - 4. You should then divide both sides by 2 to isolate the y variable, yielding the equation y = 2a - 2.

Choice B is incorrect. This equation may be the result of not dividing both terms by 2 when isolating y in the equation 2y = 4a - 4.

Choice C is incorrect. This equation may result from not distributing the 4 when multiplying 4 and (a - 1).

Choice D is incorrect. This equation may result from solving 2y = 4a - 4 for a, yielding $a = \frac{1}{2}y + 1$. A misunderstanding of the meaning of variables may have resulted in switching the variables to match the answer choice.

15

Which of the following is equal to (14 - 2i)(7 + 12i)? (Note: $i = \sqrt{-1}$)

- A) 74
- B) 122
- C) 74 + 154i
- D) 122 + 154i

Content: Additional Topics in Math

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When you have an equation in which two fractions are set equal to each other, cross-multiply in order to eliminate the fractions.

Key: D

Objective: You must apply the distributive property on two complex binomials and then simplify the result.

Explanation: Choice D is correct. Applying the distributive property to multiply the binomials yields the expression $98 + 168i - 14i - 24i^2$. The note in the question reminds you that $i = \sqrt{-1}$, therefore, $i^2 = -1$. Substituting this value into the expression gives you 98 + 168i - 14i - (-24), and combining like terms results in 122 + 154i.

Choice A is incorrect and may result from a combination of errors. You may not have correctly distributed when multiplying the binomials, multiplying only the first terms together and the second terms together. You may also have used the incorrect equality $i^2 = 1$.

Choice B is incorrect and may result from a combination of errors. You may not have correctly distributed when multiplying the binomials, multiplying only the first terms together and the second terms together.

Choice C is incorrect and results from misapplying the statement $i = \sqrt{-1}$.

16

The graph of y = (2x - 4)(x - 4) is a parabola in the xy-plane. In which of the following equivalent equations do the x- and y-coordinates of the vertex of the parabola appear as constants or coefficients?

A)
$$y = 2x^2 - 12x + 16$$

B)
$$y = 2x(x-6) + 16$$

C)
$$y = 2(x-3)^2 + (-2)$$

D)
$$y = (x - 2)(2x - 8)$$

Content: Passport to Advanced Math

Key: C

Objective: You must be able to see structure in expressions and equations and create a new form of an expression that reveals a specific property.

Explanation: Choice C is correct. The equation y = (2x - 4)(x - 4) can be written in vertex form, $y = a(x - h)^2 + k$, to display the vertex, (h, k), of the parabola. To put the equation in vertex form, first multiply: $(2x-4)(x-4) = 2x^2 - 8x - 4x + 16$. Then, add like terms, $2x^2 - 8x - 4x + 16 = 2x^2 - 12x + 16$. The next step is completing the square.

$$y = 2x^{2} - 12x + 16$$

$$y = 2(x^{2} - 6x) + 16$$

$$y = 2(x^{2} - 6x + 9 - 9) + 16$$

$$y = 2(x^{2} - 6x + 9) - 18 + 16$$

$$y = 2(x - 3)^{2} - 18 + 16$$

 $y = 2(x-3)^2 - 2$

Isolate the x^2 term by factoring Make a perfect square in the parentheses Move the extra term out of the parentheses Factor inside the parentheses Simplify the remaining terms

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Multiply complex numbers in the same way you would multiply binomials (by "FOIL"ing).

Remember that $i = \sqrt{-1}$ and that $i^2 = -1$.

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While you may be asked to write the equation of a parabola in vertex form, sometimes simply knowing the form that displays the coordinates of the vertex will suffice, saving you precious time. Therefore, the coordinates of the vertex, (3, -2), are only both revealed in choice C. Since you are told that all of the equations are equivalent, simply knowing the form that displays the coordinates of the vertex will save all of these steps — this is known as "seeing structure in the expression or equation."

Choice A is incorrect; it displays the *y*-value of the *y*-intercept of the graph (0, 16) as a constant.

Choice B is incorrect; it displays the *y*-value of the *y*-intercept of the graph (0, 16) as a constant.

Choice D is incorrect; it displays the *x*-value of one of the *x*-intercepts of the graph (2, 0) as a constant.

17

If $a^{-\frac{1}{2}} = x$, where a > 0 and x > 0, which of the following equations gives a in terms of x?

A)
$$a = \frac{1}{\sqrt{x}}$$

B)
$$a = \frac{1}{x^2}$$

C)
$$a = \sqrt{x}$$

D)
$$a = -x^2$$

Content: Passport to Advanced Math

Key: B

Objective: You must demonstrate fluency with the properties of exponents. You must be able to relate fractional exponents to radicals as well as demonstrate an understanding of negative exponents.

Explanation: Choice B is correct. There are multiple ways to approach this problem, but all require an understanding of the properties of exponents. You may rewrite the equation as $\frac{1}{\sqrt{a}} = x$ and then proceed to solve for a, first by squaring both sides, which gives $\frac{1}{a} = x^2$, and then by multiplying both sides by a to find $1 = ax^2$. Finally, dividing both sides by x^2 isolates the desired variable.

Choice A is incorrect and may result from a misunderstanding of the properties of exponents. You may understand that a negative exponent can be translated to a fraction but misapply the fractional exponent.

Choice C is incorrect and may result from a misunderstanding of the properties of exponents. You may recognize that an exponent of $\frac{1}{2}$ is the same as the square root but misapply this information.

Choice D is incorrect and may result from a misunderstanding of the properties of exponents. You may recognize that raising a to the power of $\frac{1}{2}$ is the

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Know the exponent rules and practice applying them. This question tests several of them:

1)
$$a^{-b}$$
 can be written as $\frac{1}{a^b}$

2)
$$a^{\frac{1}{2}}$$
 is the same as \sqrt{a}

3)
$$\sqrt{a^2} = a$$

4) To eliminate a radical from an equation, as in $\frac{1}{\sqrt{a}} = x$, square both sides of the equation.

same as taking the square root of *a* and, therefore, that *a* can be isolated by squaring both sides. However, you may not have understood how the negative exponent affects the base of the exponent.

18

If
$$y = x^3 + 2x + 5$$
 and $z = x^2 + 7x + 1$, what is $2y + z$ in terms of x?

- A) $3x^3 + 11x + 11$
- B) $2x^3 + x^2 + 9x + 6$
- C) $2x^3 + x^2 + 11x + 11$
- D) $2x^3 + 2x^2 + 18x + 12$

Content: Passport to Advanced Math

Key: C

Objective: You must substitute polynomials into an expression and then simplify the resulting expression by combining like terms.

Explanation: Choice C is correct. Substituting the expressions equivalent to y and z into 2y + z results in the expression $2(x^3 + 2x + 5) + x^2 + 7x + 1$. You must apply the distributive property to multiply $x^3 + 2x + 5$ by 2 and then combine the like terms in the expression.

Choice A is incorrect and may result if you correctly found 2y in terms of x but did not pay careful attention to exponents when adding the expression for 2y to the expression for z. As a result, you may have combined the x^3 and x^2 terms.

Choice B is incorrect and may result if you failed to distribute the 2 when multiplying $2(x^3 + 2x + 5)$.

Choice D is incorrect and may result from finding 2(y + z) instead of 2y + z.

19

Which of the following is equal to $\sin(\frac{\pi}{5})$?

- A) $-\cos\left(\frac{\pi}{5}\right)$
- B) $-\sin(\frac{\pi}{5})$
- C) $\cos\left(\frac{3\pi}{10}\right)$
- D) $\sin\left(\frac{7\pi}{10}\right)$

Content: Additional Topics in Math

Key: C

Objective: You must understand radian measure and have a conceptual understanding of trigonometric relationships.

PRACTICE AT



Don't worry if you missed this question, there are several ways to make a careless mistake.

Always be methodical when doing calculations or simplifying expressions, and use your test booklet to perform the steps in finding your answer.

Explanation: Choice C is correct. Sine and cosine are related by the equation $\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$. Therefore, $\sin\left(\frac{\pi}{5}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$, which reduces to $\cos\left(\frac{3\pi}{10}\right)$.

Choice A is incorrect and may result from a misunderstanding about trigonometric relationships. You may have thought that cosine is the inverse function of sine and therefore reasoned that the negative of the cosine of an angle is equivalent to the sine of that angle.

Choice B is incorrect and may result from a misunderstanding of the unit circle and how it relates to trigonometric expressions. You may have thought that, on a coordinate grid, the negative sign only changes the orientation of the triangle formed, not the value of the trigonometric expression.

Choice D is incorrect. You may have confused the relationship between sine and cosine and erroneously added $\frac{\pi}{2}$ to the given angle measure instead of subtracting the angle measure from $\frac{\pi}{2}$.

20



The semicircle above has a radius of r inches, and chord \overline{CD} is parallel to the diameter \overline{AB} . If the length of \overline{CD} is $\frac{2}{3}$ of the length of \overline{AB} , what is the distance between the chord and the diameter in terms of r?

- A) $\frac{1}{3}\pi r$
- B) $\frac{2}{3}\pi r$
- C) $\frac{\sqrt{2}}{2}r$
- D) $\frac{\sqrt{5}}{3}r$

Content: Additional Topics in Math

Key: D

Objective: This problem requires you to make use of properties of circles and parallel lines in an abstract setting. You will have to draw an additional line in order to find the relationship between the distance of the chord from the diameter and the radius of the semicircle. This question provides an opportunity for using different approaches to find the distance required: one can use either the Pythagorean theorem or the trigonometric ratios.

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Question 20 is a particularly challenging question, one that may require additional time to solve. Be careful, however, not to spend too much time on a question. If you're unable to solve a question in a reasonable amount of time at first, flag it in your test booklet and return to it after you have attempted the rest of the questions in the section.

Explanation: Choice D is correct. Let the semicircle have center O. The diameter \overline{AB} has length 2r. Because chord \overline{CD} is $\frac{2}{3}$ of the length of the diameter, $CD = \frac{2}{3}(2r) = \frac{4}{3}r$. It follows that $\frac{1}{2}CD = \frac{1}{2}\left(\frac{4}{3}\right)r$ or $\frac{2}{3}r$. To find the distance, x, between \overline{AB} and \overline{CD} , draw a right triangle connecting center O, the midpoint of chord \overline{CD} , and point C. The Pythagorean theorem can then be set up as follows: $r^2 = x^2 + \left(\frac{2}{3}r\right)^2$. Simplifying the right-hand side of the equation yields $r^2 = x^2 + \frac{4}{9}r^2$. Subtracting $\frac{4}{9}r^2$ from both sides of the equation yields $\frac{5}{9}r^2 = x^2$. Finally, taking the square root of both sides of the equation will reveal $\frac{\sqrt{5}}{3}r = x$.

Choice A is incorrect. If you selected this answer, you may have tried to use the circumference formula to determine the distance rather than making use of the radius of the circle to create a triangle.

Choice B is incorrect. If you selected this answer, you may have tried to use the circumference formula to determine the distance rather than making use of the radius of the circle to create a triangle.

Choice C is incorrect. If you selected this answer, you may have made a triangle within the circle, using a radius to connect the chord and the diameter, but then may have mistaken the triangle for a 45-45-90 triangle and tried to use this relationship to determine the distance.

PRACTICE AT



Advanced geometry questions may require you to draw shapes, such as triangles, within a given shape in order to arrive at the solution.